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Implementation of Bose-Einstein interference effects in Monte Carlo generators

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Abstract. An efficient method of calculating weights implementing the Bose-Einstein interference effects is presented. The usefulness of the method is demonstrated for the events generated by the JETSET/PYTHIA code to describe the *UA1* proton-antiproton data at 630 GeV. A good description of data is achieved with a reasonable value of the Gaussian width of a two-particle weight factor, which is the only free parameter in our calculations.

1 Introduction

Recently various methods of imitating the Bose-Einstein interference effects in Monte Carlo generators have been discussed [1-3]. This problem became acute with the advent of new data on WW production, since quite conflicting estimates of the W mass shifts resulting from these effects have been published [4, 5].

It has been shown that the most commonly used method of momenta shifting (employed in the new versions of JETSET [6]) does not reproduce the input shape of the Bose-Einstein correlation factor, even after improvements of the original procedure [3]. Thus modifying this shape to fit the data we cannot hope to learn much about the space-time evolution of the production process.

On the other hand, the method attributing weights to generated events, much better justified theoretically [7], as recently discussed in more detail by Białas and Krzywicki [8] is not easy to implement. The factorial increase of the number of terms to be calculated with multiplicity makes impossible the direct application of the method for high energies. Only recently Wosiek [9] has indicated a possible scheme shortening such calculations. Symmetrizing separately the particles from each hemisphere, as proposed by Haywood [1], introduces the unknown bias against the effect for slow particles (in the CM). In addition, this procedure does not remove the fundamental difficulties but only shifts them to higher multiplicities. Another problem of the weight method is the serious distorsion of the multiplicity distribution, since weights enhance the high multiplicity tail [2].

Solution to these problems has been proposed recently by Jadach and Zalewski [5], who reduce the number of terms replacing the original Białas - Krzywicki formula by an approximation based on the clustering algorithm. The initial average multiplicity is restored by rescaling the weights with a simple cV^n factor.

Another method of implementing weights, based on the genuine symmetrization of amplitudes has been suggested a long time ago by Andersson and Hoffman [10] and recently presented in detail by Andersson and Ringner [11]. This method, however, is specific to the "JETSET based" generators. Thus it seems reasonable to discuss further the possible realizations of the more general Białas-Krzywicki method.

In this note we propose a rapidly convergent approximation procedure for the Białas - Krzywicki formula which avoids prohibitive increase in the computing time with multiplicity. We present the method in the next section and then apply it to the description of the *UA1* data [10]. The results are encouraging. We regard them as a first succesful generator – independent and reasonably sound method for implementing BE effects in the MC generators.

We hope to use the same method to clarify the problem of the W mass shifts in hadronic decays.

2 Calculating weights for multiparticle states

According to the Białas - Krzywicki prescription, the symmetrization of amplitudes required by the Bose-Einstein statistics may be approximated by generating unsymmetrized distributions and correcting them *a posteriori* by the multiplicative weights attributed to each event. Such a weight is a sum over all permutations of identical particles of the products of two particle weight factors $w_{iP(i)}$ calculated for the pair of momenta (of i-th particle and the particle, which occupies the i-th place in the permutation $P\{i\}$).

$$W(n) = \sum_{\{P(i)\}} \prod_{i=1}^{n} w_{iP(i)}. \qquad (1)$$

Since all factors are positive and $w_{ii} = 1$, the resulting weight is not smaller than one (a contribution from identity permutation). One may correct results to keep, e.g., the average number of particles fixed; we return to this point later.

Since most of the particles detected in experiments are pions, the final weight should be actually given by a product of weights calculated separately for positive, negative and neutral pions. In fact, the BE interference for neutral particles is not observable (apart from the possible effects for hard photons [13]): neutral pions decay before detection, and for the resulting photons the effective source size is so big that the BE effects must be negligible for momentum differences above a few eV. However, the procedure should not change the observable correlations between the numbers of charged and neutral pions. Therefore weights for all signs of pions must be taken into account.

Thus in principle the only arbitrary factor is the function of the difference of two momenta $w_{ij}(p_i - p_j)$. It is natural to try as a first guess the Gaussian function of four-momentum difference squared

$$w_{ij} = e^{(p_i - p_j)^2/2\sigma^2} \qquad (2)$$

which is motivated by a commonly used experimental parametrization of BE effects.

Of course, different components of momentum difference squared may be multiplied by different coefficients, and the shape may be modified. In this note we do not discuss these possibilities. Therefore the only parameter is a Gaussian half-width of the distribution σ.

We have tried first to implement this prescription generating by the JETSET/PYTHIA program [6,14] the samples of 10^5 events of $\bar{p}p$ collisions at 10 and 30 GeV CM energies. For each event for each permutation the product of two-particle weight factors (with $\sigma = 0.14$ GeV which corresponds to 1 fm radius of the Fourier transform of the weight (2)) is computed and all contributions are added to calculate the weight for the event which is used to produce distributions (to be compared with those for all weights equal one).

Unfortunately, for more than ten pions of a given sign the calculations become prohibitively long. This does not happen at 10 GeV, but already at 30 GeV there are more then hundred events with such multiplicities. To get any results we had to exclude them from standard weight calculations (attributing to each of them the same value of the weight as obtained for the previously generated event). This removes, however, the fluctuations which are most interesting for the investigation of very short range correlations.

Thus here we have separated the sum of all the n! permutations into terms where only the permutations which change places of exactly K particles are taken into account:

$$w = \sum_K w^{(K)}. \qquad (3)$$

The higher terms in this expansion correspond to configurations where many particles have approximately the same momenta, which is very unlikely. These terms for $K < 6$ are

$$w^{(0)} = 1;$$

$$w^{(1)} = 0;$$

$$w^{(2)} = \sum_{i=1}^{n-1} \sum_{j>i} (w_{ij})^2;$$

$$w^{(3)} = 2 \sum_{i=1}^{n-2} \sum_{j>i} \sum_{k>j} w_{ij} w_{jk} w_{ki};$$

$$\begin{aligned} w^{(4)} = \sum_{i=1}^{n-3} \sum_{j>i} \sum_{k>j} \sum_{l>k} [&2 w_{ij} w_{ik} w_{jl} w_{kl} \\ &+ 2 w_{ij} w_{il} w_{jk} w_{kl} + 2 w_{ik} w_{il} w_{jk} w_{jl} \\ &+ (w_{il} w_{jk})^2 + (w_{ij} w_{kl})^2 + (w_{ik} w_{jl})^2]; \end{aligned}$$

$$\begin{aligned} w^{(5)} = 2 \sum_{i=1}^{n-4} \sum_{j>i} \sum_{k>j} \sum_{l>k} \sum_{m>l} [&(w_{ij})^2 w_{lk} w_{ml} w_{km} \\ &+ (w_{ik})^2 w_{jl} w_{ml} w_{jm} + (w_{il})^2 w_{jk} w_{jm} w_{km} \\ &+ (w_{im})^2 w_{jk} w_{kl} w_{jl} + (w_{jk})^2 w_{il} w_{lm} w_{im} \\ &+ (w_{jl})^2 w_{ik} w_{km} w_{im} + (w_{jm})^2 w_{ik} w_{kl} w_{il} \\ &+ (w_{kl})^2 w_{ij} w_{jm} w_{im} + (w_{lm})^2 w_{ij} w_{jk} w_{ik} \\ &+ (w_{km})^2 w_{ij} w_{jl} w_{il} + w_{ij} w_{jk} w_{kl} w_{lm} w_{im} \\ &+ w_{ik} w_{jl} w_{km} w_{jm} w_{il} + w_{il} w_{ij} w_{kl} w_{jm} w_{km} \\ &+ w_{ij} w_{ik} w_{jl} w_{lm} w_{km} + w_{ik} w_{im} w_{jk} w_{jl} w_{lm} \\ &+ w_{il} w_{jl} w_{jk} w_{km} w_{im} + w_{ij} w_{ik} w_{kl} w_{lm} w_{jm} \\ &+ w_{ij} w_{il} w_{lm} w_{jk} w_{km} + w_{ij} w_{im} w_{jl} w_{km} w_{kl} \\ &+ w_{ik} w_{il} w_{jk} w_{lm} w_{jm} + w_{ik} w_{im} w_{jm} w_{jl} w_{kl} \\ &+ w_{il} w_{im} w_{kl} w_{jk} w_{jm}]. \end{aligned} \qquad (4)$$

To check the method we have first compared the results from the full sum of permutations (1) with the results from the sum (3) cut at $K = 4$ at 10 GeV. Both programs give the same results within a few permille accuracy for all investigated distributions. Of special interest is the "BE ratio", defined for the pair of identical pions as a function of $Q = \sqrt{-(p_1 - p_2)^2}$

$$c_2(Q) = \frac{\int d^3p_1 d^3p_2 \rho_2(p_1, p_2) \delta[Q - \sqrt{-(p_1 - p_2)^2}]}{\int d^3p_1 d^3p_2 \rho_1(p_1) \rho_1(p_2) \delta[Q - \sqrt{-(p_1 - p_2)^2}]} \times \frac{< n >^2}{< n(n-1) >}. \qquad (5)$$

Without weights it is rather flat and close to one, if we normalize separately the numerator and the denominator of (5) to the same number of entries (which is achieved by the second factor in (5)). Including weights produces a maximum at smallest Q^2 with the height about 2 (i.e. one unit above the value at large Q^2) and a width σ' about 0.15. Thus we reproduce satisfactorily the shape assumed for the two-particle weight factor.

At 30 GeV we do not have, as noted above, the results of full symmetrization; for 174 events of the highest multiplicity the weights of the previously generated events were attributed (still, this program requires 10 times more computing time than the program with no more than 4 momenta symmetrized!). The multiplicity distribution for two programs differs slightly in the tail, although average multiplicities are quite similar: 10% and 11% higher than without weights. The peak at low Q^2 in the ratio of distributions exceeds slightly 2 and looks similar in both programs.

After this exercise we started generating events at 630 GeV, the energy of the *UA1* experiment. For the calculations of weights we used the same value of σ as before. We did not have now the possibility to estimate the results of full symmetrization (the highest multiplicities of one sign pions exceeded 40 in the 10^5 events sample). Thus we have checked first that cutting the series (3) at $K = 3$ and at $K = 4$ we get quite similar shapes of the Q^2 spectra, although the normalization is significantly different. Including the term with $K = 5$ we change even less all the distributions. Thus we feel that cutting the series (3) at $K = 5$ we get a reliable estimate of the results for Q^2 spectra from the weight method (up to the possible change of normalization).

This may seem surprising if we remember that our approximation does not take into account, e.g., the contribution from such a simple configuration as three pairs of very close (pairwise) momenta. Indeed, in this case there is a contribution from a permutation of 6 elements. However, the full contribution of such a configuration to the sum (1) is equal 1+3+3+1 = 8 (from permutations moving 0, 2, 4 and 6 elements, respectively) and our approximation counts all but the last term in this sum. We have checked that for all reasonably probable configurations our approximation seems similarly satisfactory.

The distribution of weights at 630 GeV is much broader than at previous energies and has a long tail (up to the values of a few hundreds). Consequently, the multiplicity distribution is significantly changed by the weighting. Since the JETSET/PYTHIA parameters were fitted to reproduce inclusive experimental data without weights, the change, e.g., of the average multiplicity induced by weights should be compensated by the proper refitting procedures. Instead we have applied a simple method of multiplying weights by an extra cV^n factor, where n is the number of pions, and c and V are constants fixed by the requirements to restore the original number of events and the original average multiplicity. We return later to the details of this procedure. Such a rescaling of weights does not change significantly the shape of Q^2 distributions. The BE ratio reflects mainly the assumed shape of the two-particle weight (plus 1): for larger σ it is wider and starts to increase above 2 for smallest Q^2.

The procedure seems to produce too high a value of the BE ratio for smallest Q^2. As already noted, it is about twice the value for small Q^2, whereas in most of the data it is only by some 50% higher. Let us note that we are using the old version of the *UA1* data [10] with rather arbitrary normalization, and we attempt to describe them only down to the lower limit of Q^2 about 0.01 GeV^2. In later publications of UA1 [13] the BE ratio is shown to increase above 2 for lower Q^2. Since, however, this increase is still a subject of controversy and anyway cannot be described by a Gaussian shape, we do not discuss it here.

To explain why the BE ratio does not increase up to the value of 2, one may invoke some coherent component, but a more obvious effect (which also lowers the BE ratio) is the existence of longer living resonances. Pions coming from their decay are effectively "born" more than 10 fm from the collision point. Thus the Gaussian width parameter in a two-particle weight for these pions should by smaller by an order of magnitude, which allows practically to neglect their contribution to be BE effect in the experimentally accessible Q^2 range. Therefore the Białas-Krzywicki weights should be calculated taking into account only the permutations of momenta of pions produced directly, or resulting from the decay of the widest resonances.

This is achieved easily if the procedure calculating weights is called before the decay of long-living resonances, i.e. in the same place, where the original LUBOEI procedure was called. We have rewritten correspondingly our program separating the procedure LWBOEI (called directly from JETSET, and calculating for each event a weight as a product of weights for positive, negative and neutral "direct" pions) from the master program (calculating distributions with and without weights). This procedure is available from authors as a FORTRAN file. The results are now (after rescaling the weights, as desribed above) quite similar to the data and may be brought to even better agreement by fitting the only free parameter - the Gaussian half-width of a two particle weight σ. We discuss this comparison with data in more detail in the next section.

3 Results and comparison with data

We have generated 10^5 events of $p\overline{p}$ minimum bias collisions at 630 GeV CM energy by the default version of the PYTHIA/JETSET generator [6,14]. For each event the weight factor was calculated by taking the 4-momenta of "direct" pions of each sign, calculating for them a matrix of two - particle weights w_{ij} according to (2) with $\sigma = 0.14$ GeV, and then the weight w as a series (3) cut at $K = 4$ or 5. As already noted, the event weight is a product of weight factors for all three kinds of pions.

Since the charged pion multiplicity distribution (used to fit the default values of the parameters with all weights equal 1) is strongly affected by weights, we rescale the weight factors to restore the original average multiplicity. To this end we multiply the weights by a cV^n factor, where n is the number of all "direct" pions of the event, and c and V are calculated from the comparison of the original and weighted multiplicity distribution. This is done by assuming that the original multiplicity distribution of "direct" pions may be well approximated by the negative binomial formula, i.e. that the NBD parameters $\overline{n}$ and $1/k$ are given by the experimental values of $< n >$ and $< n(n-1 > / < n >^2 -1$. If with the weights we get a new average multiplicity $< n' >$, the original value may then be restored by rescaling the weights with

$$V = \frac{< n > (< n' > +k)}{< n' > (< n > +k)} \tag{6}$$

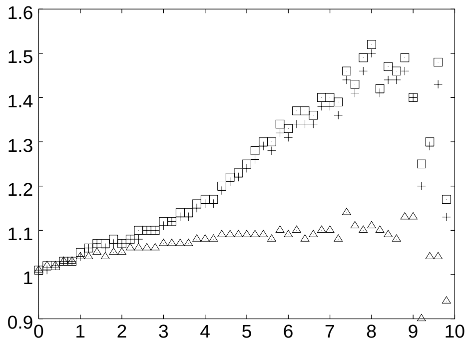

Fig. 1. The "BE ratio" (5) for positive pions as a function of $x = ln_2(1GeV^2/Q^2)$. *Triangles*, *crosses* and *squares* correspond to series (3) cut at K = 0 (no BE effect), 4 and 5, respectively

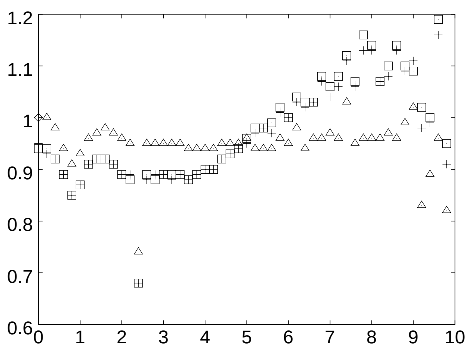

Fig. 2. The "double ratio" of ratios (5) for ++ and +- pion pairs as a function of $x = ln_2(1GeV^2/Q^2)$. *Triangles*, *crosses* and *squares* correspond to series (3) cut at K = 0 (no BE effect), 4 and 5, respectively

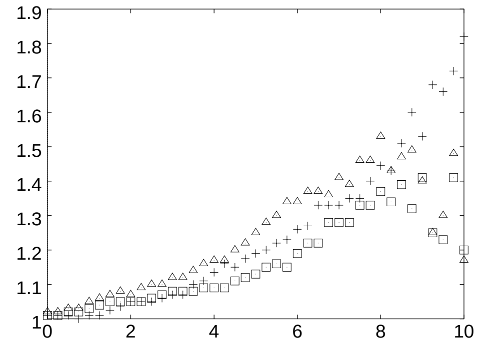

Fig. 3. The "BE ratio" (5) for positive pions as a function of $x = ln_2(1GeV^2/Q^2)$. *Crosses* represent the *UA1* data [10], *triangles* and *squares* correspond to the series (3) cut at $K = 5$ with $\sigma = 0.14\ GeV$ and $\sigma = 0.1\ GeV$, respectively

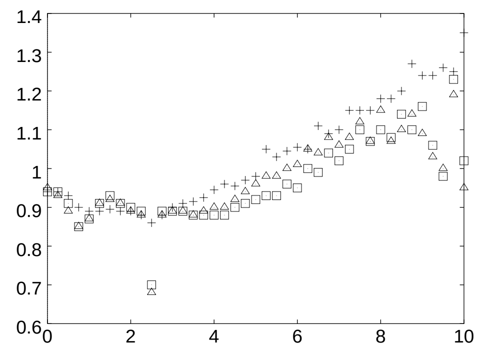

Fig. 4. The "double ratio" of ratios (5) for ++ and +- pion pairs as a function of $x = ln_2(1GeV^2/Q^2)$. *Crosses* represent the *UA1* data [10], *triangles* and *squares* correspond to the series (3) cut at $K = 5$ with $\sigma = 0.14\ GeV$ and $\sigma = 0.1\ GeV$, respectively

and

$$c = \frac{[1 + (1 - V) < n' > /k]^k}{< w >}, \qquad (7)$$

where $< w >$ is the average value of weights before rescaling. We have checked that this procedure restores indeed the original average multiplicity with accuracy of few percent. If this accuracy is not satisfactory, the quantities c and V can be estimated by direct minimization of differences between the multiplicity distributions without weights and with the measured weights, respectively. On the other hand, the BE ratios are little affected by rescaling (only the normalization, which is anyway mainly a matter of convention, changes by a few percent).

In Fig. 1 we present the "BE ratio" (5) for pairs of positive pions as a function of $x = ln_2(1GeV^2/Q^2)$ for the events from the original PYTHIA/JETSET generator (without weights) and from our prescription with series (3) cut at $K = 4$ and $K = 5$. In Fig. 2 we show the "double ratio", i.e. the ratio of (5) for pairs of positive- and unlike sign pions for the same events.

We see that without weights both ratios are very close to one and depend weakly on Q^2 (the dip in the double ratio at $x = 2.5$, *i.e.* $Q^2 = 0.17GeV^2$ is the reflection of K_s^0 in unlike sign pairs, and the wider dip at lower x comes from ρ). Our prescription produces a clear increase of both ratios at low Q^2, and the difference between two choices of K_{max} are almost negligible. Thus we believe that cutting the series (3) at $K = 5$ we approximate very well the results with full formula for the weights (1), which would require an unreasonably long computation time (even for supercomputers) when multiplicity exceeds 20.

In Fig. 3 and Fig. 4 we compare our results obtained for $K \leq 5$ and two values of σ (0.14 and 0.1 GeV) with the *UA1* data [12] normalized as in (5).

Let us stress here once more that we use on purpose an old version of the data, which seemed to be well described by a Gaussian shape of the ratio (5) for $Q^2 > 0.01GeV^2$ ($x < 8$) . Possible strong enhancement of this ratio for lower values of Q^2 discussed in more detail in the later *UA1* papers [15], which seems to signal a non-Gaussian shape, would require a modification of the shape of the two-particle weight w_{ij}. Since here our purpose is merely to prove the reliability of our method we do not want to enter this problem. In most of the hadroproduction data such low values of Q^2 are anyway not available and the data seem to be relatively well fitted by a Gaussian.

We see that the data for the BE ratio up to x = 8 are bracketed by the results for two values of σ, corresponding to the source radius of 1 and 1.4 fm. We do not attempt to fit σ here more precisely, since our purpose is only to demonstrate the applicability of the method presented above.

4 Conclusions and outlook

We have shown that the weight method of implementing the Bose-Einstein interference effects in Monte Carlo generators may be applied effectively to describe the data. The prohibitive increase of computing time with multiplicity is avoided by an approximation, in which only the selected class of terms ($K \leq K_{max}$ in (3)) out of all $n!$ contributions is taken into account. We show that already for $K_{max} = 4$ and $K_{max} = 5$ the results are almost the same, which suggests that they approximate well those for the full series.

The change of multiplicity distributions induced by weights is compensated by simple rescaling, equivalent to refitting of the original MC parameters. Using the simple one parameter Gaussian form of two-particle weight factor we describe reasonably well the *UA1* data. Of course, in the future detailed investigations it is just the shape of the weight factor which should be fitted to the data, and we hope to learn from it more about the space-time evolution of multihadron production processes.

To support the applicability of our method we investigated other effects of the weighting procedure. In particular, we checked how the weights change single particle distributions. The slope of the transverse momentum distribution increases by a few percent, and the pseudorapidity distribution becomes also slightly narrower. Obviously the changes are stronger for larger σ, when the weight distribution is broader. However, all these effects are rather small and may be easily compensated by small corrections in the values of free parameters in JETSET.

We have also checked that the effects in BE ratios are quite similar for the low- and high multiplicity event samples. The more detailed comparison with data (taking into account the experimental trigger conditions and cuts) is now in progress and looks quite promising.

There are many directions in which these results should be extended. One should check if the weight method can describe higher order BE effects and semi-inclusive ratios. The possibility of non-Gaussian and non-symmetric weight factors should be investigated. Other hadroproduction processes should be compared with $p\overline{p}$ collisions. We hope to learn soon whether our method is reliable enough to apply it confidently to the estimate of W mass shifts in the four-jet final states of the $e^+e^- \to W^+W^-$ collisions.

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